Vacuum Energy Problem, Fundamental Length and Deformed Quantum Field Theory

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Abstract

The cosmological constant (vacuum energy) problem is analyzed within the scope of quantum theories with UV-cut-off or fundamental length. Various cases associated with the appearance of the latter are considered both using the Generalized Uncertainty Relations and the deformed density matrix,previously introduced in the author's works. The use of the deformed density matrix is examined in detail. It is demonstrated that, provided the Fischler-Susskind cosmic holographic conjecture is valid, the Vacuum Energy Density takes a value close to the experimental one. The arguments supporting the validity of this conjecture are given on the basis of the recently obtained results on Gravitational Holography.

1 Introduction

The problem of vacuum energy is one of the key problems in a modern theoretical physics. This problem has attracted the attention of researchers fairly recently with the understanding that a cosmological constant determining the vacuum energy density is still nonzero despite its smallness. As is known, the cosmological constant Λ has been first introduced in the works of A.Einstein [1] who has used it as a antigravitational term to obtain solutions for the equations of a General Relativity (GR) in the stationary case. However, when A.Friedmann has found the solutions for GR in case of expanding Universe [2]and E.Hubble has derived an extension of the latter, A.Einstein refused from the cosmological constant calling it "his greatest error". Specifically, in his letter to H.Weyl in 1923, Einstein's comments on the discovery of

expanding Universe were something like [3]"If a quasistatic world is nonexistent the cosmological term should be dropped!" But Λ has survived due to the above-mentioned reasons. The principal problem of the cosmological constant resides in the fact that its experimental value is smaller by a factor of 10^{123} than that derived using a Quantum Field Theory (QFT).

And the theories actively developed at the present time (e.g., superstring theory, loop quantum gravity, etc.) offer a modified quantum theory including, in particular, the fundamental length at Planck's scale. The estimates of Λ obtained on the basis of these theories may be greatly differing from the initial ones derived from the standard QFT.

This paper presents a brief discussion of the relationship between the appearance of the Fundamental Length and Vacuum Energy Density Problem or Cosmological Constant Problem, from different viewpoints and with the use of various approaches. Note that the Vacuum Energy Density Problem is the main part of a more general problem – Dark Energy Problem.

2 Vacuum Energy and Generalized Uncertainty Relations

As a model, the fundamental length was introduced in the works devoted to QFT about fifty years ago. It should be noted that such a length was not necessarily associated with the Planck scales. But due to the development of a string theory, Heisenberg's uncertainty relations were modified in the 80-ies of the XX century. These new relations were called the Generalized Uncertainty Relations (GUR) [4]:

$$\Delta x \geq \frac{1}{\Delta p} + \lambda(\Delta p), \tag{1}$$

that implies a minimal length $2\sqrt{\lambda}$, where λ is on the order of the Planck area $l_p^2 \sim G$.

GUR enables one to estimate the value of Λ for several cases, in particular for the Schwarzschild-de Sitter space-time (**SdS**) with the geometry [4]

$$ds^2 = -\left(1 - \frac{2M}{r} - \frac{1}{3}\Lambda r^2\right)dt^2 + \left(1 - \frac{2M}{r} - \frac{1}{3}\Lambda r^2\right)^{-1}dr^2 + r^2 d\Omega. \tag{2}$$

These spaces are in many respects similar to Schwartzshild's black holes. To illustrate, they possess a horizon, and the notion of temperature may be

introduced for them like for the case of a black hole when the temperature T is still defined in terms of the surface gravity of horizons. Using the formula for T and entropy S in (**SdS**)derived in [4], and also the correction to these values, with the use of GUR the authors have succeeded in approaching the real estimate of the cosmological constant Λ in the case under study

$$\frac{\Lambda}{m_p^2} \sim 10^{-120}. \tag{3}$$

However, the importance of this approach is somewhat lowered as it is applicable only to the case studied or to the case of de Sitter space-time.

A more general approach with the use of GUR has been first proposed in [5]. In so doing GUR were involved into a more general context: stable deformation of Poincaré-Heisenberg algebra. Here "stable" means deformation immunity, with the retained structural constants of the associated Lie algebra. In the previous works [6] and [7] it has been shown that only the Lie algebras immune to infinitesimal deformations may be significant in physics. In [5] a deformation of of Poincaré-Heisenberg algebra is constructed stabilized Poincaré-Heisenberg algebra (SPHA). Apart from GUR, it carries three additional parameters: the length scale pertaining to the Planck/unification scale, the second length scale associated with cosmos, and a new dimensionless constant.

Note that such an extension of Poincaré-Heisenberg algebra is always leading to the noncommutative space-time, and the coordinate operators X_μ and X_ν for different μ and ν do not commute. Specifically, in the case considered in [7],[5] the appropriate commutative relation takes the form

$$[X_\mu, X_\nu] = i\ell_U^2 J_{\mu\nu}, \tag{4}$$

where $J_{\mu\nu}$ are generators of the rotation group and $\ell_U = \gamma \ell_P$. γ is a new constant: $10^{-17} \leq \gamma \leq 1$. ℓ_P as usual is a Planck scale.

By the approach developed in [5] the inclusion of the commutation relations (SPHA) may result in modification of an expression for the vacuum energy density, with its value approaching the real one. This is demonstrated taking as an illustration zero point energy for a simple harmonic oscillator [6]. It is assumed that the proposed method may be extended to give estimates for the effective value of the cosmological constant Λ.

3 Deformed Density Matrix, Holographic Principle and Vacuum Energy

As it has been repeatedly demonstrated earlier, a Quantum Mechanics of the Early Universe (Plank Scale) is a Quantum Mechanics with the Fundamental Length (QMFL)[21]–[26]. In the works [11]–[20] an approach to the construction of QMFL has been developed with the help of the deformed density matrix, the density matrix deformation $\rho(\alpha)$ in QMFL being a starting object called the density pro-matrix and deformation parameter (additional parameter) $\alpha = l_{min}^2/x^2$, where x is the measuring scale and $l_{min} \sim l_p$.

As has been indicated in [11]–[20] deformation parameter α is varying within the limits $0 < \alpha \leq 1/4$, moreover $\lim_{\alpha \to 0} \rho(\alpha) = \rho$, where ρ being the density matrix in the well-known Quantum Mechanics (QM). The explicit form of the above-mentioned deformation gives an exponential ansatz:

$$\rho^*(\alpha) = \sum_i \omega_i exp(-\alpha)|i><i|, \tag{5}$$

where all $\omega_i > 0$ are independent of α and their sum is equal to 1.

The correspondent deformed quantum field theory is defined at the non-uniform lattice in hypercube $I_{1/4}^4$ with the side $1/4$ in length and edge of $I_{1/4} = (0; 1/4)$ [16],[17]. It is designated as QFT^α. All the variables associated with the considered α - deformed quantum field theory are hereinafter denoted by the upper index $^\alpha$.

Then QFT^α will be compatible with the holographic principle, i.e. with the holographic entropy bound derived in the earlier works [8],[27].

As follows from the holographic principle, the maximum entropy that can be stored within a bounded region $\Re$ in 3-space must be proportional to the value $A(\Re)^{3/4}$, where $A(\Re)$ is the surface area of $\Re$. Of course, this is associated with the case when the region $\Re$ is not an inner part of a particular black hole. Provided a physical system contained in $\Re$ is not bounded by the condition of stability to the gravitational collapse, i.e. this system is simply non-constrained gravitationally, then according to the conventional QFT $S_{\max}(\Re) \sim V(\Re)$, where $V(\Re)$ is the bulk of $\Re$. However in Holographic Principle case, as it has been demonstrated by G. 't Hooft and R. V. Buniy and S. D. H. Hsu in [8],[27]

$$S_{\max}(\Re) \sim \frac{A(\Re)^{3/4}}{l_p{}^2}, \tag{6}$$

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And as was be shown in [28] QFT^α will be compatible with the holographic principle, i.e. with the holographic entropy bound derived in (6). In terms of the deformation parameter α the principal values of the Vacuum Energy Problem may be simply and clearly defined. Let us begin with the Schwarzschild black holes, whose semiclassical entropy is given by

$$S = \pi R_{Sch}^2/l_p^2 = \pi R_{Sch}^2 M_p^2 = \pi \alpha_{R_{Sch}}^{-1}, \tag{7}$$

with the assumption that in the formula for α $R_{Sch} = x$ is the measuring scale and $l_p = 1/M_p$. Here R_{Sch} is the adequate Schwarzschild radius, and $\alpha_{R_{Sch}}$ is the value of α associated with this radius. Then, as it has been pointed out in [29]), in case the Fischler- Susskind cosmic holographic conjecture [30] is valid, the entropy of the Universe is limited by its "surface" measured in Planck units [29]:

$$S \le \frac{A}{4}M_P^2, \tag{8}$$

where the surface area $A = 4\pi R^2$ is defined in terms of the apparent (Hubble) horizon

$$R = \frac{1}{\sqrt{H^2 + k/a^2}}, \tag{9}$$

with curvature k and scale a factors.

Again, interpreting R from (9) as a measuring scale, we directly obtain(8) in terms of α:

$$S \le \pi \alpha_R^{-1}, \tag{10}$$

where $\alpha_R = l_p^2/R^2$. Therefore, the average entropy density may be found as

$$\frac{S}{V} \le \frac{\pi \alpha_R^{-1}}{V}. \tag{11}$$

Using further the reasoning line of [29] based on the results of the holographic thermodynamics, we can relate the entropy and energy of a holographic system [31, 32]. Similarly, in terms of the α parameter one can easily estimate the upper limit for the energy density of the Universe (denoted here by ρ_{hol}):

$$\rho_{hol} \le \frac{3}{8\pi R^2}M_P^2 = \frac{3}{8\pi}\alpha_R M_P^4, \tag{12}$$

that is drastically differing from the one obtained with a naive QFT

$$\rho_{QFT}^{naive} \sim M_P^4. \tag{13}$$

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Here by ρ_{QFT}^{naive} we denote the energy Density calculated from the naive QFT [33]. Obviously, as α_R for R determined by (9) is very small, actually approximating zero, ρ_{hol} is by several orders of magnitude smaller than the value expected in QFT - ρ_{QFT}^{naive}.

In fact, the upper limit of the right-hand side of(12) is attainable, as it has been demonstrated in [34] and indicated in [29]. The "overestimation" value of r for the energy density ρ_{QFT}^{naive}, compared to ρ_{hol}, may be determined as

$$r = \frac{\rho_{QFT}^{naive}}{\rho_{hol}} = \frac{8\pi}{3}\alpha_{\mathbf{R}}^{-\mathbf{1}} = \frac{8\pi}{3}\frac{R^2}{L_P^2} = \frac{8\pi}{3}\frac{S}{S_P}, \tag{14}$$

where S_P is the entropy of the Plank mass and length for the Schwarzschild black hole. It is clear that due to smallness of α_R the value of α_R^{-1} is on the contrary too large. It may be easily calculated (e.g., see [29])

$$r = 5.44 \times 10^{122} \tag{15}$$

in a good agreement with the astrophysical data.

Naturally, on the assumption that the vacuum energy density ρ_{vac} is involved in ρ as a term

$$\rho = \rho_M + \rho_{vac}, \tag{16}$$

where ρ_M - average matter density, in case of ρ_{vac} we can arrive to the same upper limit (right-hand side of the formula(12)) as for ρ.

4 Fischler-Susskind Conjecture and Gravitational Holography

In this Section the arguments in support of the Fischler-Susskind cosmic holographic conjecture are given on the basis of the results obtained lately on Gravitational Holography.

Quite recently, T.Padmanabhan in a series of his papers [40]–[46] and some other works has convincingly demonstrated that Einstein equations may be derived from the surface term of the GR Lagrangian, in fact containing the same information as the bulk term.

And as Einstein-Hilbert's Lagrangian has the structure $L_{EH} \propto R \sim (\partial g)^2 + \partial^2 g$, in the usual approach the surface term arising from $L_{sur} \propto \partial^2 g$ has to

be canceled to get Einstein equations from $L_{bulk} \propto (\partial g)^2$ [47]. But due to the relationship between L_{bulk} and L_{sur} [42]–[44],[47], we have

$$\sqrt{-g}L_{sur} = -\partial_a \left(g_{ij} \frac{\partial \sqrt{-g}L_{bulk}}{\partial(\partial_a g_{ij})} \right) \qquad (17)$$

In such a manner one can suggest a holographic character of gravity in that the bulk and surface terms of the gravitational action contain identical information. However, there is a significant difference between the first case when variation of the metric g_{ab} in L_{bulk} leads to Einstein equations, and the second case associated with derivation of the GR field equations from the action principle using only the surface term and virtual displacements of horizons [39], whereas the metric is not treated as a dynamical variable [47]. In the case under study, it is assumed from the beginning that we consider the spaces with horizon. It should be noted that in the Fischler-Susskind cosmic holographic conjecture it is implied that the Universe represents spherically symmetric space-time, on the one hand, and has a (Hubble) horizon (9)), on the other hand. But proceeding from the results of [40]– [47], an entropy boundary is actually given by the surface of horizon measured in Planck's units of area [43]:

$$S = \frac{1}{4}\frac{A_H}{l_p{}^2}, \qquad (18)$$

where A_H is the horizon area.

Because of this, it should be noted that Einstein's equations may be obtained from the proportionality of the entropy and horizon area together with the fundamental thermodynamic relation connecting heat, entropy, and temperature [31]. In fact [40]– [47], this approach has been extended and complemented by the demonstration of holographicity for the gravitational action (see also [48]).

To sum it up, an assumption that space-time is spherically symmetric and has a horizon is the only natural assumption held in the Fischler-Susskind cosmic holographic conjecture to support its validity. Then there is a resemblance to thermodynamic systems, and one can associate the notions of temperature and entropy with them. In the case of Einstein-Hilbert gravity, it is possible to interpret Einstein's equations as the thermodynamic identity $TdS = dE + PdV$ [49].

5　Notes

I.This note is devoted to the demonstration of the fact, that in case of the holographic principle validity in terms of the new deformation parameter α in QFT^α, considered above and introduced as early as 2002 [35]–[37], all the principal values associated with the Vacuum (Dark) Energy Problem may be defined simply and naturally. At the same time, there is no place for such a parameter in the well-known QFT, whereas in QFT with the fundamental length, specifically in QFT^α it is quite natural [11, 12, 14, 16, 17, 19].

II. As indicated in [28], QFT^α (similar to the conventional QFT) conforms to the Holographic Principle, being coincident with QFT to a high accuracy in a semiclassical approximation, i.e. for $\alpha \to 0$. In this case α is small rather than vanishing. Specifically, the smallness of α_R results in a very great value of r in (14),(15). Besides, from (14) it follows that there exists some minimal entropy $S_{min} \sim S_P$, and this is possible only in QFT with the fundamental length.

III.It should be noted that this section is related to Section 3 in [38] as well as to Sections 3 and 6 in [39]. The constant L_Λ introduced in these works is such that in case under consideration $\Lambda \equiv l_\Lambda^{-2}$ is equivalent to R, i.e. $\alpha_R \approx \alpha_{l_\Lambda}$ with $\alpha_{l_\Lambda} = l_p^2/l_\Lambda^2$. Then expression in the right-hand side of (12) is the major term of the formula for ρ_{vac}, and its quantum corrections are nothing else as a series expansion in terms of α_{l_Λ} (or α_R):

$$\rho_{\text{vac}} \sim \frac{1}{l_P^4}\left(\frac{l_P}{L_\Lambda}\right)^2 + \frac{1}{l_P^4}\left(\frac{l_P}{l_\Lambda}\right)^4 + \cdots = \alpha_{l_\Lambda} M_P^4 + \alpha_{l_\Lambda}^2 M_P^4 + ... \qquad (19)$$

In the first variant presented in [38] and [39] the right-hand side (19) (formulas (12),(33) in [38] and [39], respectively)reveals an enormous additional term $M_P^4 \sim \rho_{QFT}^{naive}$ for renormalization. As indicated in the previous Section, it may be, however, ignored because the gravity is described by a pure surface term. And in case under study, owing to the Holographic Principle, we may proceed directly to (19). Moreover, in QFT^α there is no need in renormalization as from the start we are concerned with the ultraviolet-finiteness.

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